

INTERNATIONAL BACCALAUREATE
Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.19]
PERCENTAGE CHANGE - FINANCIAL APPLICATIONS

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

PERCENTAGE CHANGE

1. [Maximum mark: 8]

(a) The population of village A **increases** by 8% every year. If the population today is 1000 people find

(i) the population after 5 years;

(ii) the population 5 years ago;

(iii) the number of full years after the population will exceed 2000.

[4]

(b) The population of village B **decreases** by 8% every year. If the population today is 1000 people find

(i) the population after 5 years;

(ii) the population 5 years ago;

(iii) the number of full years after the population will fall under 500.

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2. [Maximum mark: 7]

The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.

- (a) Write down the population at the end of 1973. [1]
- (b) Find the population at the end of 2002. [3]
- (c) How long will it take for the size of the population to exceed 400 000? [3]

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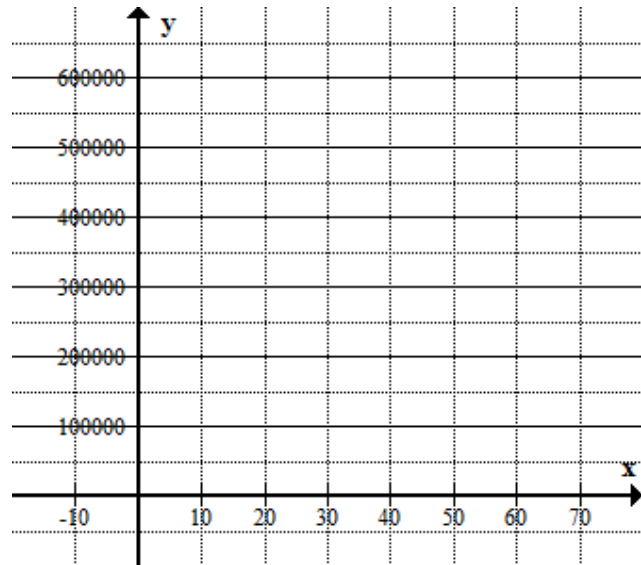
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Extra question

Sketch a graph of Population (P) in terms of years (t) after 1972
Show the information found in questions (b) and (c)



3. [Maximum mark: 4]

A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

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Extra question

Sketch a graph of Population bacteria (b) in terms of minutes (t)
Show the information found above

4. [Maximum mark: 4]

Each year for the past five years the population of a certain country has increased at a steady rate of 2.7% per annum. The present population is 15.2 million.

(a) What was the population one year ago? [2]

(b) What was the population five years ago? [2]

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Extra question

Sketch a graph of Population (P) in millions in terms of years (t)

Show the information found in questions (a) and (b)

FINANCIAL APPLICATIONS

5. [Maximum mark: 8]

An amount of \$ 10 000 is invested at an annual interest rate of 12%.

- (a) Find the value of the investment after 5 years
 - (i) if the interest rate is compounded yearly;
 - (ii) if the interest rate is compounded half-yearly;
 - (iii) if the interest rate is compounded quarterly;
 - (iv) if the interest rate is compounded monthly. [4]

- (b) The value of the investment will exceed \$ 20 000 after n full years. Calculate the minimum value of n
 - (i) if the interest rate is compounded yearly;
 - (ii) if the interest rate is compounded monthly. [4]

METHOD A: By using the formula of FV

METHOD B: By using GDC - Financial mode

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6. [Maximum mark: 7]

An initial amount of \$ 10 000 and an extra \$ 10 000 **at the end of each year** are deposited into a savings account at an annual interest rate of 5%.

- (a) Find the value of the investment
 - (i) at the end of the 1st year
 - (ii) at the end of the 2nd year
 - (iii) at the end of the 10th year [5]
- (b) Find the least number of full years for the value of investment to exceed 200 000 [2]

METHOD A: By using the formula of FV

METHOD B: By using GDC - Financial mode

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Extra question

Explain why the value of the investment at the end of the n^{th} year is given by

$$FV = 10000 \times \left(\frac{1.05^{n+1} - 1}{0.05} \right)$$

[if last payment is not included we subtract 10000]

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7. [Maximum mark: 7]

An amount of \$10 000 is deposited into a savings account at an annual interest rate of 5%. An extra amount of \$ 1 000 is deposited **at the end of each year**.

(a) Find the value of the investment

(i) at the end of the 1st year

(ii) at the end of the 2nd year

(iii) at the end of the 10th year [5]

(b) Find the least number of full years for the value of investment to exceed 50 000 [2]

METHOD A: By using the formula of FV

METHOD B: By using GDC - Financial mode

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8. [Maximum mark: 6]

An amount of \$ 10 000 is deposited into a savings account at an annual interest rate of 5% compounded **monthly**. An extra amount of \$1000 is deposited **at the end of each year**.

- (a) Find the value of the investment
 - (i) at the end of the 1st year
 - (ii) at the end of the 10th year [4]
- (b) Find the least number of full years for the value of investment to exceed 50 000 [2]

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9. [Maximum mark: 6]

An amount of \$ 10 000 is deposited into a savings account at an annual interest rate of 5% compounded **monthly**. An extra amount of \$ 100 is deposited **at the end of each month**.

- (a) Find the value of the investment
 - (i) at the end of the 1st year
 - (ii) at the end of the 10th year [4]
- (b) Find the least number of full **months** for the value of investment to exceed 50 000 [2]

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10. [Maximum mark: 9]

An amount of \$ 10 000 is deposited into a savings account at an annual interest rate of 5% (compounded yearly).

(a) Find the value of the **interest** after 1st year. [1]

If an amount of \$ 500 is **withdrawn** at the end of each year;

(b) Find the amount remained in the account after 10 years. [2]

If an amount of \$ 400 is **withdrawn** at the end of each year;

(c) Find the amount remained in the account after 10 years. [2]

If an amount of \$ 1 000 is **withdrawn** at the end of each year;

(d) Find the amount remained in the account after 10 years. [2]

(e) The last withdrawal L will be made after n years. Find the values of n and of L . [2]

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11. [Maximum mark: 7]

Ann gets a loan of € 20000 for 10 years at a nominal annual interest rate of 15 % compounded **yearly**.

- (a) Find the repayment made each **year** (correct to 2 decimal places) [3]
- (b) Find the total amount paid. [2]
- (c) Find the interest paid on the loan. [2]

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12. [Maximum mark: 7]

Bill gets a loan of € 20000 for 10 years at a nominal annual interest rate of 15 % compounded **monthly**.

- (a) Find the repayment made each **month**. (correct to 2 decimal places) [3]
- (b) Find the total amount paid. [2]
- (c) Find the interest paid on the loan. [2]

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13. [Maximum mark: 7]

Chris gets a loan of € 20000 for 10 years at a nominal annual interest rate of 15 % compounded **quarterly**.

- (a) Find the repayment made each **quarter** (correct to 2 decimal places) [3]
- (b) Find the total amount paid. [2]
- (c) Find the interest paid on the loan. [2]

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14. [Maximum mark: 7]

Dimitris gets a loan of € 20000 for 10 years at a nominal annual interest rate of r % compounded **monthly**. Terms of the loan require monthly repayments of € 300.

- (a) Find the value of r . [3]
- (b) Find the total amount paid. [2]
- (c) Find the interest paid on the loan. [2]

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15. [Maximum mark: 5]

A sum of \$ 5 000 is invested at a compound interest rate of 6.3 % per annum. The value of the investment will exceed \$ 10 000 after n full years. Calculate the minimum value of n .

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16. [Maximum mark: 6]

\$1000 is invested at the beginning of each year for 10 years.

The rate of interest is fixed at 7.5% per annum. Interest is compounded annually.

Calculate, giving your answers to the nearest dollar

- (a) how much the first \$1000 is worth at the end of the ten years; [2]
- (b) the total value of the investments at the end of the ten years. [4]

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17. [Maximum mark: 5]

\$1000 is invested at 15% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed \$3000.

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18. [Maximum mark: 8]

Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually.

- (a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc. [2]
- (b) How many complete years will it take for Michele's initial investment to double in value? [3]
- (c) What should the interest rate be if Michele's initial investment were to double in value in 10 years? [3]

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B. Paper 2 questions (LONG)

19. [Maximum mark: 18]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % compounded quarterly.

No deposit required and repayments are made each quarter.

- (a) (i) Find the repayment made each quarter.
- (ii) Find the total amount paid for the car.
- (iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % compounded monthly. Terms of the loan require a 10 % deposit and monthly repayments of €250.

- (b) (i) Find the amount to be borrowed for this option.
- (ii) Find the annual interest rate, r . [5]
- (c) State which option Bryan should choose. Justify your answer. [2]

Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

- (d) If they invest it in an account paying 0.4% interest per month and inflation is 0.1% per month, calculate the real amount of money the car dealership has received by the end of the 6 year period. [4]

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20. [Maximum mark: 16]

Themis sold his car at the beginning of 2020 and received an amount of € 7000.

- (a) Find the **real** value of Themis' amount at the end of 2020,
 - (i) given that the inflation rate is 1.1 % per year.
 - (ii) given that the inflation rate is 0.1 % per month. [3]
- (b) Find the **real** value of Themis' amount at the end of 2024,
 - (i) given that the inflation rate is 1.1 % per year.
 - (ii) given that the inflation rate is 0.1 % per month. [4]

Themis decides to invest his money at 12% per year, **compounded monthly**.

- (c) Find the amount he will receive back at the end of 2024. [2]
- (d) Find the **real** value of the money he will receive at the end of 2024 if the inflation rate is 0.1 % per month. [3]

Themis decides to invest his money at 12% per year, **compounded monthly** and withdraw € 100 at the end of each month.

- (e) Find the amount remaining in his account at the end of 2024. [2]
- (f) Find the month he will receive the last amount remaining [2]

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